Beknopte uitwerkingen

1. $$\int_0^1 \arctan x\,dx = \Big[x\arctan x\Big]_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx =$$
$$\frac{\pi}{4} - \Big[\tfrac{1}{2}\ln(1+x^2)\Big]_0^1 = \frac{\pi}{4} - \frac{1}{2}\ln 2.$$

2. $$\int_0^1 x\ln(1+x)\,dx = \Big[\tfrac{1}{2}x^2\ln(1+x)\Big]_0^1 - \int_0^1 \tfrac{1}{2}x^2\cdot\frac{1}{1+x}\,dx =$$
$$\tfrac{1}{2}\ln 2 - \tfrac{1}{2}\int_0^1\Big(\frac{x^2-1}{1+x} + \frac{1}{1+x}\Big)dx = \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\int_0^1\Big((x-1)+\frac{1}{1+x}\Big)dx$$
$$= \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\Big[\tfrac{1}{2}x^2 - x + \ln(1+x)\Big]_0^1 = \tfrac{1}{2}\ln 2 - \tfrac{1}{2}\Big(-\tfrac{1}{2} + \ln 2\Big) = \tfrac{1}{4}$$

3. $$\int x^2e^{-x}\,dx = -\int x^2\,de^{-x} = -x^2e^{-x} + \int e^{-x}\cdot 2x\,dx =$$
$$-x^2e^{-x} - \int 2x\,de^{-x} = -x^2e^{-x} - 2xe^{-x} + 2\int e^{-x}\,dx =$$
$$(-x^2-2x-2)e^{-x} + C.$$

4. a) breuksplitsen $\dfrac{1}{x(2x+1)} = \dfrac{A}{x} + \dfrac{B}{2x+1}$

$A(2x+1) + Bx = 1 \qquad A = 1,\ B = -2.$

$$\int\Big(\frac{1}{x} - \frac{2}{2x+1}\Big)dx = \ln|x| - \ln|2x+1| + C = \ln\Big|\frac{x}{2x+1}\Big| + C$$
$$\int_1^\infty \frac{1}{x(2x+1)}\,dx = \lim_{p\to\infty}\Big[\ln\frac{x}{2x+1}\Big]_1^p = \ln\tfrac{1}{2} - \ln\tfrac{1}{3} = \ln\tfrac{3}{2}$$

b) $$\int e^{-\sqrt{x}}\,dx = \int e^{-t}\cdot 2t\,dt = -\int 2t\,de^{-t} = -2te^{-t} + 2\int e^{-t}\,dt.$$

$$\boxed{\begin{array}{l}\sqrt{x} = t\\ x = t^2\\ dx = 2t\,dt\end{array}}$$

$$-2te^{-t} - 2e^{-t}$$

$$\int_0^\infty e^{-\sqrt{x}}\,dx = \int_0^\infty e^{-t}\cdot 2t\,dt = \lim_{p\to\infty}\Big[-2te^{-t} - 2e^{-t}\Big]_0^p = 2$$

$$\Big(\lim_{p\to\infty} pe^{-p} = \lim_{p\to\infty}\frac{p}{e^p} \underset{\text{l'Hospital}}{=} 0\Big)$$

5. $$\int xe^{-2x}\,dx = -\tfrac{1}{2}\int x\,de^{-2x} = -\tfrac{1}{2}xe^{-2x} + \int\tfrac{1}{2}e^{-2x}\,dx = -\tfrac{1}{2}xe^{-2x} - \tfrac{1}{4}e^{-2x}$$
$$\int_0^\infty xe^{-2x}\,dx = \lim_{p\to\infty}\Big[-\tfrac{1}{2}xe^{-2x} - \tfrac{1}{4}e^{-2x}\Big]_0^p = \tfrac{1}{4}$$

6.a) Breuksplitsen. $\int \frac{1}{x^2+x}\,dx = \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx = \ln\left|\frac{x}{x+1}\right|$

$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$.

$A(x+1) + Bx = 1 \quad A = 1,\ B = -1$

b) $\int_1^\infty \frac{1}{x^2+x}\,dx = \lim_{p\to\infty}\left[\ln\left|\frac{x}{x+1}\right|\right]_1^p = -\ln\frac{1}{2} = \ln 2.$
dus convergent.

(of $0 \le \frac{1}{x^2+x} \le \frac{1}{x^2}$ $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent en dus ook $\int_1^\infty \frac{1}{x^2+x}\,dx$
is convergent)

7. $\underline{r}(t) = \begin{pmatrix} 3\cos t \\ 3\sin t \\ 2t\sqrt{t} \end{pmatrix}$, $0 \le t \le 3$.

$\underline{r}'(t) = \begin{pmatrix} -3\sin t \\ 3\cos t \\ 3\sqrt{t} \end{pmatrix}$ $\quad |\underline{r}'(t)| = 3\sqrt{1+t}$. $\quad \int_k ds = \int_{t=0}^3 3\sqrt{1+t}\,dt = \left[2(1+t)^{\frac{3}{2}}\right]_0^3$

$= 16 - 2 = 14.$

8. $\underline{r}(t) = \begin{pmatrix} t^2 \\ \sqrt{3}t \\ t \end{pmatrix}$, $0 \le t \le \sqrt{3}$ $\quad \underline{r}'(t) = \begin{pmatrix} 2t \\ \sqrt{3} \\ 1 \end{pmatrix}$ $\quad |\underline{r}'(t)| = \sqrt{4t^2+4}$

a) $t = \frac{1}{2}\sqrt{3}$. $\quad \underline{r}'(\frac{1}{2}\sqrt{3}) = \begin{pmatrix} \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix}$ $\quad \left|\begin{pmatrix} \sqrt{3} \\ \sqrt{3} \\ 1 \end{pmatrix}\right| = \sqrt{7}$

gevraagde eenheidsvector $\left\langle \frac{\sqrt{3}}{\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{7}}, \frac{1}{\sqrt{7}} \right\rangle$.

b) $\int_0^{\sqrt{3}} \sqrt{4t^2+4}\,dt = \int_0^{\sqrt{3}} 2\sqrt{t^2+1}\,dt = \left[t\sqrt{1+t^2} + \ln\left(t + \sqrt{1+t^2}\right)\right]_0^{\sqrt{3}}$.
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$2\sqrt{3} + \ln(\sqrt{3}+2)$

9. Snijden K_1 en K_2: $\begin{cases} 2\cos t = \sqrt{2}s \quad s = \sqrt{2} \\ 2\sin t = \sqrt{2}s \quad s = \sqrt{2} \\ 2t = 2s + \frac{\pi}{2} - 2 = \frac{\pi}{2}. \end{cases}$ ~~$\rightarrow 4\cos^2 t + 4\sin^2 t = 4s^2 = 4$ $s = \pm 1$~~ $\rightarrow t = \frac{\pi}{4}, s = 1$

klopt ook met alle andere vergelijkingen.

$\underline{r}_1'(t) = \begin{pmatrix} -2\sin t \\ 2\cos t \\ 2 \end{pmatrix}$ $\quad \underline{r}_1'(\frac{\pi}{4}) = \begin{pmatrix} -\sqrt{2} \\ \sqrt{2} \\ 2 \end{pmatrix} = \underline{a}$

$\underline{r}_2'(s) = \begin{pmatrix} \sqrt{2} \\ \sqrt{2} \\ 2 \end{pmatrix} = \underline{b}$, θ is hoek tussen $\underline{a}$ en $\underline{b}$

$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|} = \frac{-2+2+4}{\sqrt{8}\cdot\sqrt{8}} = \frac{4}{8} = \frac{1}{2} \rightarrow \theta = \frac{\pi}{3}$